First Quarter Project

Pre-AP Calculus

You don't really recognize these relations when you are working with just the theory; in order to appreciate the intricacy, you need to actually handle and apply them, as we did here.

~excerpted from the concluding paragraphs of a student's math project (former student of Mr. O'B)

Instructions: You and a partner will choose one of the following six- email your choice to Mr. O'Brien ASAP (first come, first served). Some are more applicable to "real life" and others are more from the realm of pure mathematics. Are some easier than others?

You will be given some class time and some homework time to do your investigation and final write-up. You will use your graphing calculator as you investigate. Your final version **must be typed** up (I will show you how to use Google Docs to facilitate this collaborative process).

Keep in mind that there are often no single correct answers, and you will be evaluated on the basis of your reasoning, justification, and communication skills. **Effective communication of ideas** is a very important component of mathematics.

The **rubric** for evaluation is on the back of this page. Your work will count as one of the two test grades for this unit.

Rough Draft Due:___________________

Final Draft Due:___________________

Quarter 1 Project Rubric

Category	Poor	Fair	Good	Excellent
Presentation (10%) • Is the paper neat? • Is the paper typed? • Is the paper done in an orderly manner?				
Mathematical precision and completeness (50%) • Are the solutions complete? • Are the solutions correct? • Has there been a correct use of mathematical notation?				
Verbal explanations (30%) • Are the explanations correct? • Are the explanations complete and precise? • Is there correct use of grammar and spelling?				
Graphs (10%) • Are the graphs correct? • Are the graphs neat? • Are the axes labeled properly?				

General Comments:

Final Grade:________

Crickets – Nature's Thermometer

Crickets are one of nature's more interesting insects, partly because of their musical ability. In England, the chirping or singing of a cricket was once considered a sign of good luck. In China and Japan, crickets were kept in fancy cages in the house so the residents could enjoy their singing. Many of us are so used to hearing this sound on a summer evening that we would probably think that something was wrong if it were missing. The male cricket "sings" to attract the female cricket—not just to keep you up at night—by rubbing his two front wings together.[1]

An interesting fact about crickets is that their activity depends on temperature. As a result, they can be thought of as "natural" thermometers. The rate of a cricket's chirp increases as the temperature increases; it also depends on the type of cricket. So, if you know the right formula and the type of cricket you hear chirping, you can estimate the temperature by counting the chirps. Changes in humidity and different crickets of the same type also produce variations in a cricket's chirping rate. The dominant factor, however, is temperature, so formulas relating temperature to the number of chirps are fairly accurate. The following are rules for finding the temperature, in degrees Fahrenheit, for three different types of crickets.[2]

- The field cricket is the black cricket commonly found in the United States. For a field cricket, count the number of chirps in 15 sec and add 38 to obtain the temperature.

- The tree cricket is small and pale green and is usually found on trees. For this cricket, the temperature can be obtained by counting the number of chirps in 7 sec and adding 46.

- The snowy tree cricket is the species whose music is most in tune with that of the temperature since it is believed to be the most accurate. For this cricket, count the number of chirps in 14 sec and add 42.

[1] Ross E. Hutchins, Insects (Englewood Cliffs, NJ: Prentice-Hall, 1966), pp. 54–57.
[2] Lucy Clausen, Insect Fact and Folklore (New York: MacMillan, 1958), pp. 62–63.

1. Find a function relating temperature to the number of chirps by following these steps.

(a) Notice that each "rule" involves counting the number of chirps in a different predetermined period: 15 sec for the field cricket, 7 sec for the tree cricket, and 14 sec for the snowy tree cricket. The input for our function will be *number of chirps per minute*.

i. For the field cricket, if n is the number of chirps per minute, what is the number of chirps in 15 sec?

ii. For the tree cricket, if n is the number of chirps per minute, what is the number of chirps in 7 sec?

iii. For the snowy tree cricket, if n is the number of chirps per minute, what is the number of chirps in 14 sec?

(b) Translate the "rule" for each cricket into a function where the input is n, number of chirps per minute, and the output is T, temperature in degrees Fahrenheit.

(c) If each type of cricket chirps 120 times in 1 min, what are the three different temperatures?

2. In the following questions, various properties and characteristics of the functions relating temperature to the number of chirps are examined.

(a) Explain how you know, just from looking at your functions from question 1, part (b), that these equations are linear.

(b) For each function, specify a domain (inputs that are reasonable for this situation) and the corresponding range. (*Note:* Assume these functions are not valid when the temperature is above 100°F.)

(c) Graph each of the three functions on the same set of axes.

(d) Explain the physical meaning of the y-intercept in terms of temperature and number of cricket chirps.

(e) Remembering that slope is (change in output) $\div$ (change in input), explain the physical meaning of slope for these functions in terms of temperature and number of cricket chirps.

3. We now want to find functions where the input is temperature and the output is the number of chirps per minute; that is, we wish to find the inverse functions for the three functions found in question 1 part (b).

(a) For each of the three functions from question 1, part (b), solve for n in terms of T; that is, find the inverse function.

(b) What is the domain and the range of each of your inverse functions? How do they compare with the domain and the range for the three original functions?

(c) What is the slope of each of the inverse functions? How does it compare with the slope of the original functions?

(d) If it is 70°F, how many chirps will each cricket produce in 1 min?

(e) Graph the three inverse functions on the same set of axes.

(f) We wish to determine which cricket chirps at the highest rate. So, we are looking for the line with the largest y-values, or the line that is "on top."

i. Examine your graph for the temperature range from 40°F to 70°F. Notice that the line that has the highest y-values depends on where you are in this interval. Find the point of intersection for these two lines.

ii. Which cricket chirps at the highest rate and at which temperatures when $40 < T < 70$?

(g) The slope of a line tells how "steep" the line is. If you have two lines, is it always true that the line with the largest slope gives the largest output? Explain your answer. How does this relate to your answer to question 3, part (f)?

4. Suppose we wish to find temperature in degrees Celsius[3] rather than degrees Fahrenheit. For this question, consider only the field cricket rather than all three crickets.

(a) Use the function found in question 1, part (b) for the *field cricket* and convert it to a new function relating number of chirps per minute to temperature in degrees Celsius. (*Hint:* Use $T_C = \frac{5}{9}(T_F - 32)$, where T_F is temperature in degrees Fahrenheit and T_C is temperature in degrees Celsius.)

[3] This would give us a metric cricket.

(b) Convert this function to a verbal rule, such as one of those originally given, that can be used easily to convert chirps to temperature in degrees Celsius. Your rule should be written in the following form: Count the number of chirps in __ seconds and add __. Refer back to question 1, part (a), to remind yourself how to convert from words to symbols. (*Note:* For this rule to be easily used, it is necessary to round quantities to the nearest whole number.)

A Taxing Problem

The Internal Revenue Service is the department given the task of collecting taxes for the U.S. Government. There are different tax brackets for different income levels. As people make more money, they move to a higher tax bracket and the rate at which they are taxed increases. Suppose the IRS used the following piecewise function to figure taxes (i = taxable income, t = taxes owed).

$$\begin{array}{lrlll}
\text{If} & \$0 & \leq i \leq \$22,100 & \text{then} & t = 0.15i. \\
\text{If} & \$22{,}100 & < i \leq \$53,500 & \text{then} & t = 0.28i. \\
\text{If} & \$53{,}500 & < i \leq \$115,000 & \text{then} & t = 0.31i. \\
\text{If} & \$115{,}000 & < i \leq \$250,000 & \text{then} & t = 0.36i. \\
\text{If} & \$250{,}000 & < i & \text{then} & t = 0.396i.
\end{array}$$

1. (a) Sketch the graph of this piecewise function with taxable income as the input and the tax as the output.

(b) Using the given formula, calculate the taxes owed and the resulting net income (i.e. taxable income – taxes) for those with the taxable incomes shown below in Table 1.

Taxable Income	Taxes	Net Income
\$20,000		
\$22,000		
\$22,200		
\$53,000		
\$54,000		
\$114,000		
\$115,100		
\$249,900		
\$250,100		

Table 1

(c) Find a formula for the piecewise function where taxable income is the input and net income is the output. Graph this function.

(d) Consider the difference in taxable income and net income. Why is this piecewise function NOT a good model of taxation (especially for the American citizen!)?

2. The actual tax function[5] is not as drastic as the one listed at the beginning of this project. That's because the government doesn't tax *all* your income at a higher rate when you earn a certain amount. The higher rate only applies to the money you make that exceeds certain specific amounts. For example, in 1993 if you made \$25,000 in taxable income, the government charged 15% on your first \$22,100 of taxable income and 28% on \$2,900, the amount of taxable income that exceeded \$22,100.

Table 2 shows the taxes for a single person in 1993 based on the actual tax function, rounded to the nearest dollar.

Taxable Income	Tax
\$20,000	\$3,000
\$22,000	\$3,300
\$22,200	\$3,343
\$53,000	\$11,967
\$54,000	\$12,262
\$114,000	\$30,862
\$115,100	\$31,208
\$249,900	\$79,736
\$250,100	\$79,812

Table 2

(a) Give the formula used by the government to compute taxes. Remember that the first \$22,100 is taxed at a rate of 15% and only the amount over \$22,100 is taxed at a higher rate. For example, if your taxable income is \$25,000, then

$$tax = 0.15(22,100) + 0.28(25,000 - 22,100).$$

Your answer will be a piecewise function with taxable income as the input and tax as the output. The income levels and the percentage rates are the same as those given at the beginning of this project.

(b) Check your formulas by computing the taxes on the incomes given in Table 2. Your answers should match the answers given. If they do not, modify your formula or check your arithmetic.

[5] Internal Revenue Service, *Instructions for form 1040, 1993*, U. S. Government Printing Office, pp 13,24,49.

3. Throughout this project we have been using taxable income and not a person's total income. Since most people have a better understanding of someone's total income, let's look at this and how it relates to their tax. In 1993 a single person with no dependents, taking the standard deduction,[6] could subtract \$6950 from his or her total income to obtain the taxable income. This means a person earning \$26,950 would have a taxable income of \$20,000 and a tax of \$3000. Using this information do the following:

(a) Determine the function for a single person, with no dependents who is taking the standard deduction, where the input is total income and the output is tax. Your answer will be another piecewise function. Again, use the taxable income levels and percentage rates given at the beginning of this project.

(b) Graph your function from question 3(a) with a domain of \$0 to \$120,000.

(c) What properties does the graph from question 3(b) possess that your first graph, from 1(c), did not? Why does this matter?

4. There have been proposals for years of various forms of a flat tax. This is where everyone pays the same rate regardless of their income level. Suppose the federal government chose a 20% flat tax with no deductions. However, a single person with no dependents was allowed a \$10,000 personal exemption. This means that this person would not have to pay any federal income tax on the first \$10,000 of income, but would pay 20% on any amount over \$10,000.

(a) Determine the function for a single person with no dependents where the input is total income and the output is the tax based on this flat tax.

(b) Use numerical, graphical, or symbolic techniques to determine how your function from question 4(a) compares with that from question 3(a). Taxpayers at which level of income would pay more and which would pay less using the flat rate? For which method does it seem like the government will receive more tax dollars?

[6] Not all taxpayers take the standard deduction, especially those earning high incomes. These people usually can find ways to take more deductions. This is, however, a way to standardize things for this project.

Newton – A Real Swinger

A pendulum consists of an object suspended from a fixed point so that it can freely swing back and forth. The period of a pendulum (i.e. the length of time it takes to go back and forth through one complete swing) is dependent only on the length of the string that is holding the mass, a fact first discovered by Galileo.[7] Although this is absolutely true only for an ideal pendulum (i.e. one in which the mass is concentrated at a point, the string has no mass, and there is no wind resistance on the mass as it swings), properties of ideal pendulums can still be used to mathematically determine the period of less than ideal pendulums.

More than three hundred years ago Isaac Newton used a pendulum to estimate the speed of sound. This fact was first learned by one of the authors while on a tour of Cambridge University in England. The tour guide stopped at a colonnade in Neville's Court, clapped her hands and a nice echo came back with a slight delay. She explained that this was the place where Newton determined the speed of sound using an echo. He measured the length of the hallway and doubled it. This gave him the distance that the sound traveled. He then had to find the length of time between clapping his hands and hearing the echo. By dividing d, the total distance the sound traveled, by t, the time needed for the sound to travel there and back, he would be able to compute the speed of sound. This all sounded fine except for one thing. The time difference between the clap and the echo returning was less than a second. While there were clocks in Newton's day, there were no stopwatches that would measure to the accuracy needed. How did Newton measure the time? He used a pendulum. Newton knew the relationship that existed between the length of the string and the period of a pendulum. To measure the time, he varied the length of a pendulum until the period matched up with the time between the clap and its return. Through this experiment he calculated the speed of sound to be between 920 and 1085 feet per second.[8] Not too bad with such simple instruments!

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[7] Newman, James R., ed., *The Harper Encyclopedia of Science, Revised ed.*, Harper & Row, 1967, p 894.

[8] Westfall, Richard S, *Never at Rest: A Biography of Isaac Newton*, Cambridge University Press, 1980, p 456.

1. We made a slightly less than ideal pendulum with fishing line and a small metal weight. Even though we had a little trouble with a cat who thought this was a nice toy, we managed to collect the data in Table 1. We will use this data to determine a function for the period of a pendulum.

Length of String (in inches)	Number of Periods (per minute)	Length of Period (in seconds)
1	165	
3	101	
5	80	
10	57	
15	47	
20	41	
25	37	
30	33.5	
35	31	
40	29.5	
45	27	
50	26.5	

Table 1

(a) Complete Table 1 by computing the time (in seconds) that it takes for one period of the pendulum. Round your answer to two decimal places.

(b) Graph the data from Table 1 with length of string on the horizontal axis and length of the period on the vertical axis. Sketch a smooth curve to connect the points. Is your graph linear? If not, what is the shape of your graph? What does this tell you about the function for the period of a pendulum in terms of the length of the string?

(c) The formula for the length of time of the period of a pendulum, P, is of the form $P = c\sqrt{l}$ where c is some constant and l is the length of string.

i. Is this consistent with the shape of your graph in question 1(b)? Why or why not?

ii. Complete Table 2 by using the data from Table 1 and dividing the length of the period (in seconds) by the square root of the length of the string to show the value of c for each case. Use these values to give an approximate value for the constant c. Explain how you obtained your approximation.

Length of String (in inches)	Length of Period (in seconds)	c
1		
3		
5		
10		
15		
20		
25		
30		
35		
40		
45		
50		

Table 2

2. The formula for the period of an ideal pendulum is

$$P = \frac{2\pi}{\sqrt{g}}\sqrt{l} \tag{1}$$

where P is the period of the pendulum, l is the length of the string, and g is the acceleration due to gravity.

(a) Estimate the acceleration of gravity using your approximation of c.

(b) The acceleration of gravity is approximately equal to 386 in/sec^2. How does this compare to your approximation to g found in the previous question? How do you account for the discrepancy?

(c) The acceleration of gravity is not the same everywhere on earth. For example, on a mountain the value of g would be less. How would this affect the period of a pendulum? Justify your answer.

For the rest of the project, use $P = \frac{2\pi}{\sqrt{g}}\sqrt{l}$ where $g = 386$ in/sec^2.

3. Wall clocks are often constructed so the pendulum has a period of one second, while grandfather clocks are often constructed so the pendulum has a period of two seconds.

(a) If a clock were to have a pendulum that had a period of one second, how long should the pendulum be?

(b) What is the length of a clock's pendulum that has a period of two seconds? Is this length twice that of the one-second pendulum? What does this say about the relationship between the length of the string and the period of the pendulum?

4. You should now realize that the accuracy of a pendulum clock is dependant on the length of the pendulum. A clock could have a pendulum cut incorrectly during construction or, through a change in temperature, a metal pendulum could shrink or expand.

(a) Suppose you were building a clock with a 6 inch pendulum.

i. If you accidently made the pendulum $\frac{1}{4}$ of an inch too long, how would that affect the period? Would your clock run fast or slow? By how much?

ii. How would making the pendulum $\frac{1}{4}$ inch too long affect the accuracy of the clock after 30 days?

(b) Would the clock be more or less accurate if you made the same $\frac{1}{4}$ inch error on a 12 inch pendulum? Explain.

Rolling Along

There are many objects in a household that come in rolls such as film, video tape, plastic wrap, paper towels, and toilet paper. In an effort to generate sales and to increase the perceived value of the product, some companies are manufacturing larger rolls of these items. Toilet paper, for example, can now be purchased in "double" rolls. Are these created by merely putting twice as much paper on the roll, or does something else need to be done? Just what is meant by the word "double?" In this project, we will answer these questions by investigating the relationship between the length of a roll of toilet paper when it is unrolled and the radius of the roll.

Looking under the bathroom sink, one of the authors found a package of a popular brand of toilet paper (or bathroom tissue as the company calls it). On the package, it states that there are 280 sheets per roll and each sheet is 4.5 inches by 4.5 inches. This means that if the paper was unrolled, the total length would be $280 \times 4.5 = 1260$ inches. Doing some measuring, we found that the radius of the cardboard core is $\frac{13}{16}$ of an inch and the total radius, from the center of the core to the outside of the roll, is $2\frac{1}{8}$ inches. This data is summarized below.

Total length:	$L = 1260$ in.
Core radius:	$r = \frac{13}{16}$ in.
Total radius:	$R = 2\frac{1}{8}$ in.

1. Answer the following questions, predicting what you think will occur.

(a) If the length of the roll is doubled, do you think the total radius of the roll will also double? Why or why not?

(b) If a graph was constructed with the total length of the roll, L, on the horizontal axis and the total radius, R, on the vertical axis, would it look most like (a), (b), or (c) in Figure 1? Explain your answer.

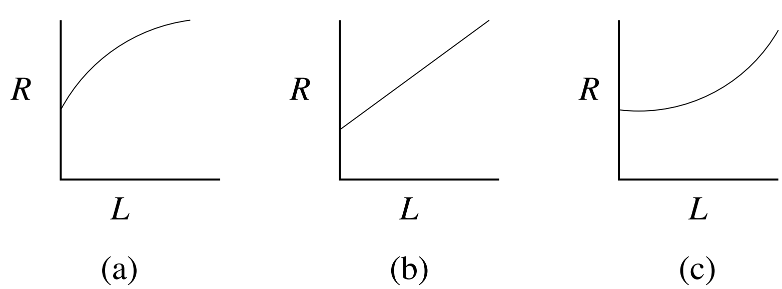

Figure 1

2. To help us understand these "double" rolls, we will derive a function that relates the total length of the paper to the total radius of the roll. We will begin by deriving a function that relates the cross-sectional area of the roll to the total radius. We will then convert this function to one that relates the total length to the total radius.

(a) Determine a function where R, the total radius of the roll, is the input and A, the cross-sectional area of the roll, is the output. Use Figure 2 as a guide. Let the total radius, R, be a variable while the core radius remains constant at $\frac{13}{16}$ inches.

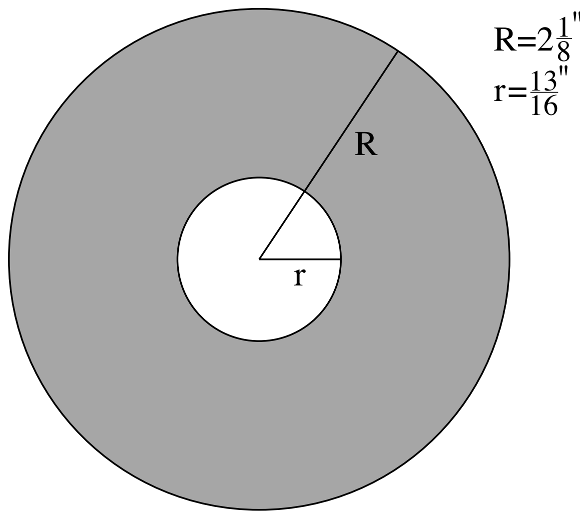

Figure 2

(b) Find the inverse of your function by solving for R. Your input should now be cross-sectional area and your output should be total radius.

(c) Suppose you were to unroll the entire roll of toilet paper. Imagine looking at this extremely long trail of toilet paper from the *side*, i.e. from the same perspective as in Figure 2 except the toilet paper is unrolled. You would see what appears to be a line. However, even though the paper is very thin, it has a thickness so what you are seeing is not really a line but rather a very thin rectangle whose length is the length of the unrolled paper and whose height is the thickness of the paper. The area of this rectangle is the same as the cross-sectional area of the paper when it is rolled up as shown in Figure 2. We will use this idea to rewrite your function from part(b) so that the input becomes the total length of the paper, L, instead of the cross-sectional area, A.

i. Using your function from 2(a), compute the cross-sectional area of the original roll shown in Figure 2 to the nearest hundredth of a square inch.

ii. When the paper is unrolled, the cross-sectional area remains the same. Using the fact that the length of the paper is 1260 inches, determine the thickness of the paper to 6 decimal places.[9]

iii. Assume L will vary while the thickness (your answer from part ii) remains constant. Write a function where the length of the paper, L, is the input and the cross-sectional area, A, is the output.

iv. Substitute what A is equal to in part iii for A in your function from 2(b). The input should now be the total length, L, and the output should still be the total radius, R.

(d) Graph your function from question 2(c)iv. Is it similar to what you predicted in question 1(b)?

3. Suppose that after college you find your dream job with the Read-A-Roll toilet paper company. Okay its not your dream job, but your mother-in-law, Rita, owns the company. Read-A-Roll reproduces famous novels directly on the roll of toilet paper which allows for readily available reading material right in the bathroom. Being a chemistry major, you are hired to determine the best way to put the ink on the rolls so it won't rub off. You also have taken many mathematics classes during your college career, so Rita chooses you to help her solve a problem.

(a) Your mother-in-law has some long novels that she wants to use, but she needs to know if they will fit on a roll. Using your function from question 2(c)iv, determine the largest roll that will fit in a typical holder. We measured a toilet paper holder and found that the roller had a diameter of $\frac{7}{8}$ of an inch and there was a clearance of 2 inches between the roller and the wall. (See Figure 3.) Determine the length of the longest roll that will fit in this holder. Will a double roll (a roll with twice the length) fit?

[9] This will preserve four significant digits.

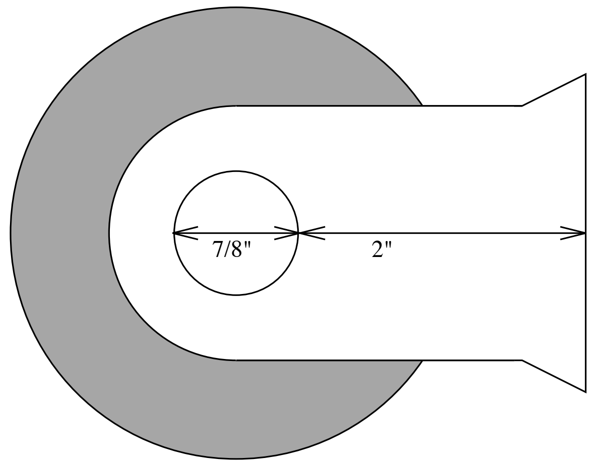

Figure 3

(b) You should have found that a true double roll will not fit in this holder. However, so called "double rolls" are manufactured and sold. What could a company, like Read-a-Roll, do differently so that a "double roll" will fit in a typical holder?

(c) Letting the core radius change (either larger or smaller), and without changing the thickness of the paper or the size of the sheets, design a truly double toilet paper roll (a roll with twice the length) that will fit on the holder shown in Figure 3.[10] Draw a picture of your new roll with the new dimensions included, and explain why it will fit on a typical holder. Be careful! The function you obtained in 2(c)iv includes $\frac{13}{16}$, which was the original core radius. This is now a variable. [Hint: The thickness of the roll of paper, $R-r$, has to be less than or equal to 2 inches.]

[10]It is possible to do this, and since your mother-in-law's dream is to use *War and Peace* for one of her novels, you are going to have to figure this out!

The Amazing Golf-O-Meter

Professional golfers take great care in determining the distance from their golf ball to the green. This is because they have a good idea how far they can hit the ball with each club, so knowing the distance pays off. Golf courses do a variety of things to help golfers determine these distances. Some have elaborate books that give a number of distances from the green to certain objects on the hole. Other golf courses have nothing at all, causing golfers to guess the distance they need to hit the ball. Because of this, there is a device available in golf accessory catalogs that claims to quickly and easily measure the distance to the flag stick. It goes by different names depending on the manufacturer, but we will simply refer to it as a golf-o-meter. In this project, you will learn how to make your own custom fit golf-o-meter and about a flaw that you may encounter in commercial ones.

A simple golf-o-meter consists of the graph in Figure 1 printed on a small piece of clear plastic. You stand looking towards the flag on the green and hold this piece of plastic up at arms length. You then move the graph so that the image of the bottom of the flag stick is on the horizontal axis and the top just touches the graph. The distance to the flag is then read on the horizontal axis.

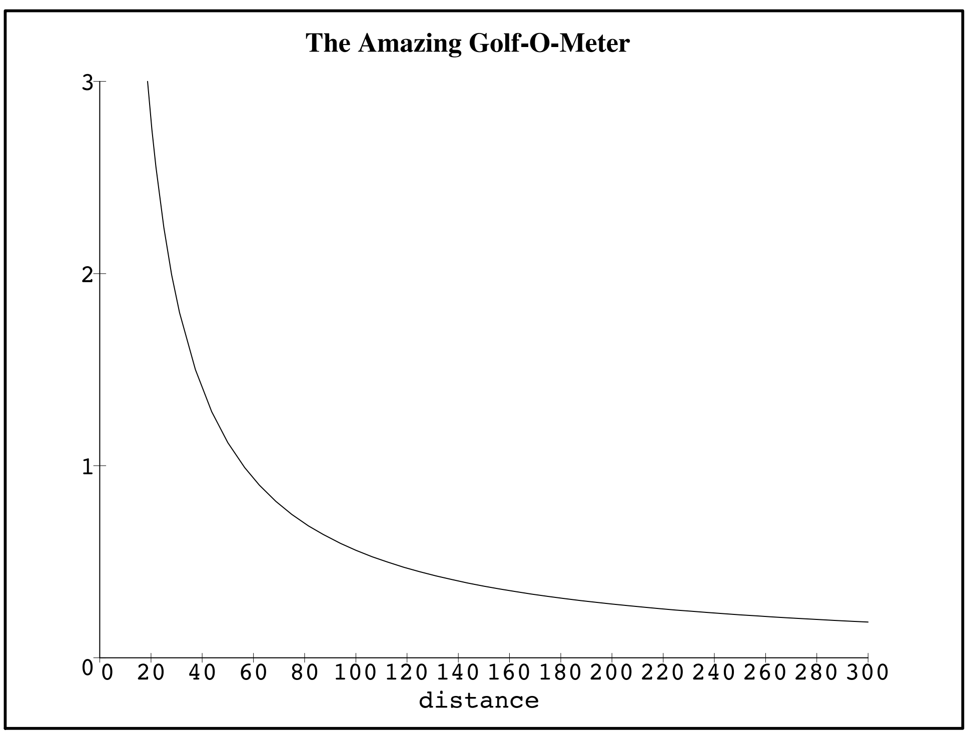

Figure 1

1. To make a standard golf-o-meter, assume that the distance from your eye to the end of your outstretched arm is 2 feet and the height of the flag stick on your golf course is 7 feet. When you look through your measuring device, the triangles shown in Figure 2 are formed.

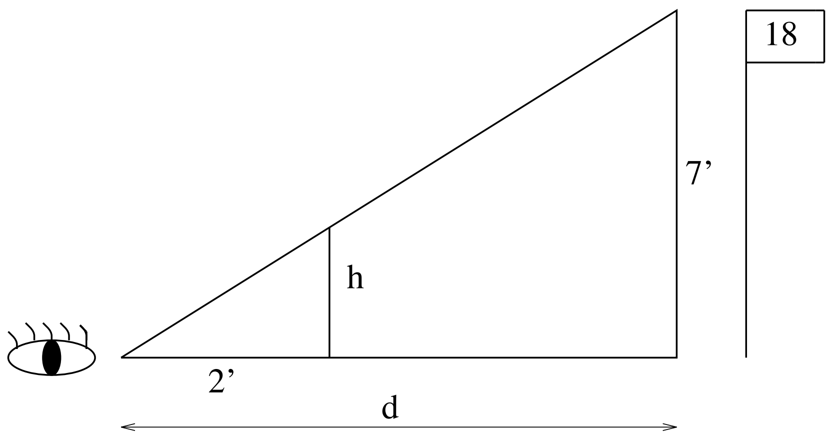

Figure 2

(a) Using Figure 2, find a function such that your input, h, is the apparent height of the flag stick as you look at it through your golf-o-meter and the output, d, is the distance you are away from the flag stick.

(b) Graph your function with a domain of 0 to 0.5 feet (reasonable answers for how tall the flag stick appears compared to the scale at the end of your outstretched arm) and with a range of 0 to 900 feet.

(c) There is a problem with using your golf-o-meter in the way the graph is currently set up. The input is the height of the flag stick, h, and this is on the horizontal axis. This won't work on our graph, because when using your golf-o-meter, the image of the flag stick is not horizontal, but is vertical. This means the observed height of the flag stick needs to be on the vertical axis, even though we think of it as the input. There is nothing wrong with doing this, it just runs counter to the way we normally think of graphs. By changing this, the horizontal axis will now be the distance to the green, d, and the vertical axis will now be h. This will also be handy since the image of the flag stick will line up with the numbers representing the distance, thus making the golf-o-meter easier to read. To switch things around, solve your equation for h, the observed height of the flag stick and graph it with d on the horizontal axis and h on the vertical axis. [Note: Make sure you change the domain and range as well.]

(d) The distance of your outstretched arm and the height of the flag stick were both given in feet. This means the function you obtained has its input, the distance from the flag stick, as well as the output, the height of the curve, both in feet. However, golf course distances are always given in yards, and the height of the curve is quite small making feet a cumbersome unit of measure. It would be more appropriate if it were in inches. Convert your function from question 1(c) so that the input is in yards and the output is in inches.

(e) Graph your modified function from 1(d) with a domain of 0 to 300 yards. This time, however, when you put it on paper, make sure that your vertical axis is scaled properly. For example, at the point on the graph where the h-coordinate is one inch, that point on the graph should be exactly one inch from the horizontal axis. Congratulations, you now have a your own golf-o-meter! To use it, reproduce the graph on a clear piece of plastic.

2. Not all golf courses have flag sticks that are 7 feet high. Our golf-o-meter, however, was constructed under this assumption. This means our golf-o-meter will be inaccurate on courses with flag sticks that aren't 7 feet.[11] Suppose you take your golf-o-meter to a course that has flag sticks that are 6.5 feet tall. Let's check out the accuracy.

(a) Derive the function for a golf-o-meter where the input, d, is in yards, and the output, h, is in inches, for a flag stick that is 6.5 feet tall.

(b) Complete Table 1. Use your function from 1(d) for the second column and your function from 2(a) for the third column. Subtract your two answers for the fourth column. Since the flag stick is really 6.5 feet but the reading for the golf-o-meter is for a 7 foot flag stick, there is error. Subtracting the two columns gives you that error.

h (inches)	7 ft distance (yards)	6.5 ft distance (yards)	error (yards)
0.2			
0.4			
0.6			
0.8			
1.0			

Table 1

[11] The distance from your eye to the end of your outstretched arm can also vary, and thus cause more inaccuracy.

(c) Using your numerical data, you should notice that this error is not constant, but is dependent on your distance from the flag. In other words, the error is a function of distance.

i. Use your numerical data to look for a pattern between the distance read on the golf-o-meter (the 7 ft distance column) and the error of this reading (the error column). Use this pattern to predict what the function will be when the input is the incorrect distance that is given by the 7' golf-o-meter and the output is the error.

ii. Symbolically manipulate your formulas to show that your predicted function is the actual function. To do this, remember that the error is the difference between the 7' golf-o-meter reading and the 6.5' golf-o-meter reading. This function has to be written such that the input is d, not h.

iii. Should this error be added to or subtracted from the distance you read on your 7' golf-o-meter? Explain your answer.

(d) Let's determine if this error is significant. If you were a golfer, you would be most apt to use a device like the golf-o-meter if you were between 50 and 250 yards away from the flag stick. Golf clubs with the lower numbers will allow you to hit the ball farther. For each number lower, you can hit the ball about fifteen yards farther. For example, if you hit a seven-iron about 150 yards, you will probably hit a six-iron about 165 yards. Taking all of this into account, do you think the error in using a 7' golf-o-meter on a course with 6.5' flag sticks is significant? Explain your answer.

Crossroads

Have you ever wondered how traffic engineers determine how long a traffic signal should stay yellow before it turns red? If it is yellow for too short of a time, a vehicle could get caught in a zone where there is neither enough time to brake safely before the light nor enough time to drive through the intersection. If it is yellow for too long, drivers will slow down and stop, thinking the light will soon turn red, and traffic will be held up. There is a critical point, shown in Figure 1, where, if you are ahead of the point, you should drive through the intersection and if you are behind the point, you should stop.[12] There is a little room for error around this point, but not much. In this project, we will find the distance from this point to the light.

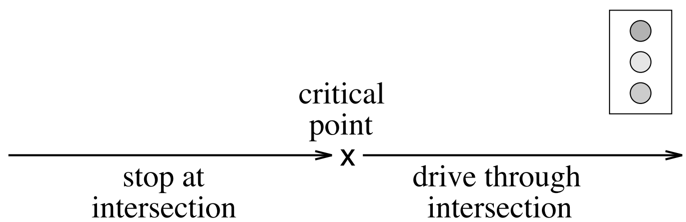

Figure 1

One way of looking at this involves using projected speeds of vehicles and information about the intersection itself. As an example, we will use the intersection at River Avenue and Eighth Street in Holland, Michigan. The speed limit on River Avenue is 30 mph (which equals 44 feet per second), so we will assume that our vehicle is going that speed. The width of the intersection is 70 feet. We will assume the reaction time (the time it takes you to apply the brake from when you see the light turn yellow) is one second. We will also assume that a safe breaking deceleration is 10 feet per second per second (which is an acceleration of -10 ft/sec^2).[13]

The first thing we want to do is find the distance our vehicle needs in order to stop. To do this we need an equation relating distance to velocity and acceleration which we will get by manipulating and combining two equations. In these equations, a = acceleration, v_f = final velocity, v_0 = initial velocity, t = time, and d = distance traveled. We will use the fact that acceleration is change in velocity divided by time, or

$$a = \frac{(v_f - v_0)}{t}. \tag{1}$$

[12] Eisenkraft, Arthur and Kirkpatrick, Larry, "Stop on Red, Go on Green..." Quantum, Jan/Feb 1994 pp 34-36.

[13] We determined these numbers through experimentation with one author at the controls of a car while breaking at 30 mph (without spilling his coffee!) and another author on a stopwatch.

Next, we need to know that average velocity is distance traveled divided by time (this is a form of the equation $d = rt$ which you should have seen before), or

$$\frac{(v_0 + v_f)}{2} = \frac{d}{t}. \tag{2}$$

From our earlier data, we have:

Initial velocity, v_0	=	44 ft/sec
Width of intersection	=	70 ft
Reaction time to brake	=	1 sec
Acceleration of braking, a	=	-10 ft/sec^2

1. Let's consider our car at the corner of Eighth Street and River Avenue and determine how long the light should remain yellow.

(a) Since we will be braking to a complete stop, the final velocity in our equations will be zero. Using this, combine equations (1) and (2) to get $d = \frac{-v_0^2}{2a}$.

(b) Once the brake is applied, how much distance will it take for our car to stop?

(c) How much total distance will it take for our car to stop once the light turns yellow, including reaction time? This is the distance from the beginning of the intersection to the critical point.

(d) From this critical point, how much time is needed for the front of our car to safely (maintaining a speed of 44 ft/sec) make it to the far side of the intersection? This will be the minimum time needed for a yellow light.

(e) The yellow light at River and Eighth stays on for 4.2 seconds. Is this what you obtained? If not, why might a difference occur?

2. Let's consider a more general situation. Assume the intersection is still 70 ft wide, the reaction time is still one second, and the acceleration of braking is still -10 ft/sec^2. Let the initial velocity vary. The final velocity, v_f, will continue to be zero since we are interested in stopping the car.

(a) Find the function whose input is the initial speed of the vehicle, v_0, in feet per second, and whose output is the time needed for a yellow light, t, in seconds. Notice that this function has three terms. One of these involves the reaction time, one involves the acceleration of braking, and one involves the width of the intersection. Identify each of these terms.

(b) Graph the function you obtained. Where is the function increasing and where is it decreasing? Recall the three terms that you identified in part (a). Explain how each of these terms is effected by the velocity of the car and how that has an effect on whether the function is increasing or decreasing.

(c) Explain, in terms of the physical situation, why the function is sometimes decreasing and sometimes increasing. Use your analysis of the three terms in your explanation.

(d) It seems reasonable that people who install lights have a manual with a chart showing the various yellow light setting for different speed zones. Complete Table 1 by giving the recommended yellow light time for speeds from 25 mph to 55 mph.

mph	ft/sec	yellow light time length
25	36.7	
30		
35		
40		
45		
50		
55		

Table 1

3. Suppose that the traffic signal at River Avenue and Eighth Street in Holland was knocked out and needs to be replaced. This job naturally falls to Herm, the city's Director of Traffic Signals and Street Sweeping. Thinking he could save a buck, Herm ordered the new signal from the mail order discount giant Traffic Signals 'R Us. Herm has a problem. These signals come preset for the amount of time the yellow light is on. (That is why they can sell them so cheaply). The new signal is preset to have a 6 second yellow light. Herm realizes that there are basically two options – return the light or find an intersection that requires a 6 second yellow light. Realizing that the first option will cost additional postage (something Herm avoids whenever possible), Herm turns to his assistant (that would be you) to investigate the situation and to find the speed (or speeds) in mph for which a six second yellow light is appropriate. Assume all of the other information (width of intersection, etc.) is the same as in the previous example. Write a memo to Herm that reports your findings.